OCR
Oxford Cambridge and RSA

Wednesday 8 June 2016 – Morning

AS GCE MATHEMATICS

4732/01 Probability & Statistics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4732/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 The table shows the probability distribution of a random variable X.

x	1	2	3	4
$P(X = x)$	0.1	0.3	0.4	0.2

(i) Find $E(X)$ and $Var(X)$. **[5]**

(ii) Three values of X are chosen at random. Find the probability that X takes the value 2 at least twice. **[3]**

2 **(i)** The table shows the amount, x, in hundreds of pounds, spent on heating and the number of absences, y, at a factory during each month in 2014.

Amount, x, spent on heating (£ hundreds)	21	23	19	15	14	5	2	10	9	20	18	23
Number of absences, y	23	25	18	18	12	10	4	9	11	15	20	26

$n = 12$ $\Sigma x = 179$ $\Sigma x^2 = 3215$ $\Sigma y = 191$ $\Sigma y^2 = 3565$ $\Sigma xy = 3343$

(a) Calculate r, the product moment correlation coefficient, showing that $r > 0.92$. **[3]**

(b) A manager says, 'The value of r shows that spending more money on heating causes more absences, so we should spend less on heating.' Comment on this claim. **[1]**

(ii) The months in 2014 were numbered 1, 2, 3, …, 12. The output, z, in suitable units was recorded along with the month number, n, for each month in 2014. The equation of the regression line of z on n was found to be $z = 0.6n + 17$.

(a) Use this equation to explain whether output generally increased or decreased over these months. **[1]**

(b) Find the mean of n and use the equation of the regression line to calculate the mean of z. **[3]**

(c) Hence calculate the total output in 2014. **[2]**

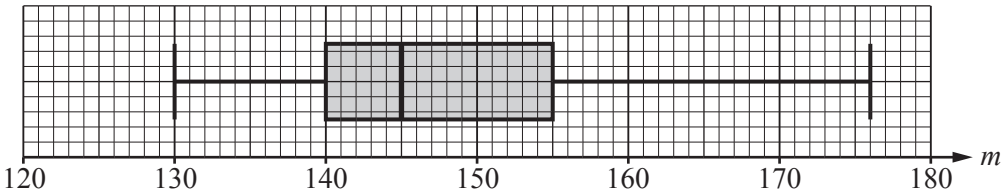
3 The masses, m grams, of 52 apples of a certain variety were found and summarised as follows.

$$n = 52 \qquad \Sigma(m - 150) = -182 \qquad \Sigma(m - 150)^2 = 1768$$

(i) Find the mean and variance of the masses of these 52 apples. **[5]**

(ii) Use your answers from part **(i)** to find the exact value of Σm^2. **[3]**

The masses of the apples are illustrated in the box-and-whisker plot below.



(iii) How many apples have masses in the interval $130 \leqslant m < 140$? **[2]**

(iv) An 'outlier' is a data item that lies more than 1.5 times the interquartile range above the upper quartile, or more than 1.5 times the interquartile range below the lower quartile. Explain whether any of the masses of these apples are outliers. **[3]**

4 In this question the product moment correlation coefficient is denoted by r and Spearman's rank correlation coefficient is denoted by r_s.

(i) The scatter diagram in Fig. 1 shows the results of an experiment involving some bivariate data.

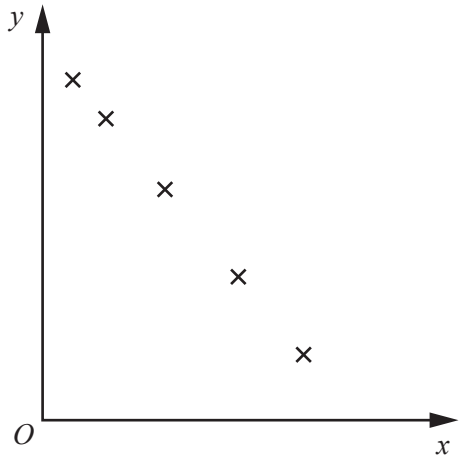


Fig. 1

Write down the value of r_s for these data. **[1]**

(ii) On the diagram in the Answer Booklet, draw five points such that $r_s = 1$ and $r \neq 1$. **[2]**

(iii) The scatter diagram in Fig. 2 shows the results of another experiment involving 5 items of bivariate data.

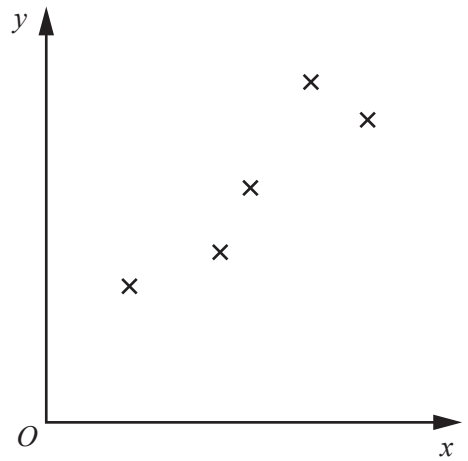


Fig. 2

Calculate the value of r_s. **[5]**

5 **(i)** A random variable X has the distribution B(25, 0.6). Find

(a) $P(X \leqslant 14)$, **[1]**

(b) $P(X = 14)$, **[2]**

(c) $\text{Var}(X)$. **[2]**

(ii) A random variable Y has the distribution B(24, 0.3). Write down an expression for $P(Y = y)$ and evaluate this probability in the case where $y = 8$. **[2]**

(iii) A random variable Z has the distribution B(2, 0.2). Find the probability that two randomly chosen values of Z are equal. **[3]**

6 **(a)** Find the number of ways in which 12 people can be divided into three groups containing 5 people, 4 people and 3 people, without regard to order. **[3]**

(b) The diagram shows 7 cards, each with a letter on it.

A	A	A	B	B	R	R

The 7 cards are arranged in a random order in a straight line.

(i) Find the number of possible arrangements of the 7 letters. **[2]**

(ii) Find the probability that the 7 letters form the name BARBARA. **[1]**

The 7 cards are shuffled. Now 4 of the 7 cards are chosen at random and arranged in a random order in a straight line.

(iii) Find the probability that the letters form the word ABBA. **[3]**

7 On average Marie scores a goal on 20% of her shots. The variable random X is the number of shots Marie takes, up to and including her first goal.

(i) State two conditions needed for X to have a geometric distribution. **[2]**

(ii) Assuming these conditions are satisfied, find the probability that

(a) $X = 3$, **[2]**

(b) $X < 10$, **[2]**

(c) $9 < X < 20$. **[3]**

The probability that Nadine scores a goal on any shot is 0.3. Marie and Nadine independently take shots in turn, with Marie shooting first. The winner is the first one to score two goals.

(iii) Find the probability that

(a) Marie wins on her second shot, **[2]**

(b) Nadine wins on her second shot. **[3]**

Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to ≥ 3sfs, ISW for later rounding
Penalise over-rounding only once in paper.

Question			Answer	Mk	Guidance	
1	i		Σxp = 2.7 oe	M1 A1	≥ 3 correct terms	÷ 4 or ÷ 10 etc M0A0
			Σx^2p (= 8.1) - "2.7"^{2} = 0.81 oe	M1 M1 A1 **[5]**	≥ 3 correct terms dep +ve result	(x - '2.7') ≥ 3 correct terms M1 $\Sigma(x$ - '2.7')^2p ≥ 3 correct terms M1 ÷ 4 or ÷ 10 etc M0M0 A0
	ii		$0.3^2 \times 0.7 \times 3 + 0.3^3$ or 0.189 + 0.027 oe = $\frac{27}{125}$ or 0.216	M2 A1 **[3]**	or $0.3^2 \times 0.7 + 0.3^3$ M1 or $0.3^2 \times 0.7 \times 3$ or 0.189 oe M1 0.3^2 or 0.09: M0 unless clearly part of correct method $0.3^2\times0.1\times3+0.3^2\times0.4\times3+0.3^2\times0.2\times3 + 0.3^3$ M2 "×3" omitted 3 times or ≥ 2 terms correct M1	1 - ($^3C_1 \times 0.3 \times 0.7^2 + 0.7^3$) M2 or 1 - ($0.3 \times 0.7^2 + 0.7^3$) M1 SC: M1 for $0.3^2\times0.6\times3+0.3^3$ or 1 - ($^3C_1 \times 0.3 \times 0.6^2 + 0.6^3$) $\times^3C_1$or $\times^3C_2$ instead of ×3 is OK throughout
Total				**8**		
2	i	a	$S_{xx} = 3215 - \frac{179^2}{12}$ (= 544.9167 or $\frac{6539}{12}$) $S_{yy} = 3565 - \frac{191^2}{12}$ (= 524.9167 or $\frac{6299}{12}$) $S_{xy} = 3343 - \frac{179\times191}{12}$ (= 493.9167 or $\frac{5927}{12}$) $r = \frac{"493.9167"}{\sqrt{"544.9167"\times"524.9167"}}$ = 0.924 (3 sf) or 0.9235.....	M1 M1 A1 **[3]**	Correct sub in a correct S formula Correct sub in 3 correct S formulae and a correct r formula Must see ≥ 3 sf	Correct ans, (≥3 sf) no wking, M1M1A1 Ignore any comparison with 0.92
	i	b	Correlation does not imply causation or It depends on the time of year or Both depend on a third variable or There could be other factors	B1 **[1]**	Any answer which implies or is equiv to any of these	Allow without context. Ignore incorrect comments

Question			Answer	Mk	Guidance		
	ii	a	'Increased' and Positive gradient or positive coeff of *n* or 'Output goes up by 0.6 each month' Both needed	 B1 **[1]**	'Increased' and values of *z* shown as follows: at least 6 values or 1st and last values or 1st, or 2nd or 3rd or 4th **and** 9th or10th or 11th or 12th ie 17.6 or 18.2 or 18.8 or 19.4 **and** 22.4 or 23 or 23.6 or 24.2	'Increased' and 'Value of 0.6*n* increases as *n* increases'	NOT: 'Increased' and 'Value of *z* incr *as n* incr' '*z* incr as no. of mths incr
	ii	b	$\bar{n}$ = 6.5 or $\frac{78}{12}$ oe seen $\bar{z}$ = 0.6×'6.5' + 17 alone, eg not ÷12 or 17 = $\bar{z}$ - 0.6×'6.5' oe $\bar{z}$ = 20.9	B1 M1 A1 **[3]**	or (0.6×1+17+0.6×2+17+ . . .0.6×12+17)÷12 oe or '250.8'÷12 M1 ft their '6.5' only if comes from ....÷12 cao	Long method, all correct terms seen and ÷12 M1 NB ans 20.9 may not score the B1	
	ii	c	Total output = "20.9" × 12 251 (3 sf)	M1 A1f **[2]**	or 0.6×1+17+0.6×2+17+ . . .0.6×12+17 oe or eg $\frac{88}{5}+\frac{91}{5}+\ldots+\frac{121}{5}$ oe ft their (ii)(b)	Long method, all correct terms seen Not ISW, eg 25100 scores A0, even if 251 seen	
Total				**10**			
3	NB in (i) and (ii) 1768 + 150^2 × 52 = 1171768 is incorrect and scores no marks in either part, except possible ft in (ii) .						
3	i		$\frac{-182}{52}$ or –3.5 seen or implied Mean = 150 – "3.5" = 146.5 or 147 $\frac{1768}{52}-("-3.5")^2$ alone, eg not if + 150 = 21.75 or 21.8	B1 M1 A1 M1 A1 **[5]**	or $\frac{-182+150\times52}{52}$ or $\frac{7800-182}{52}$ B1M1 Allow within $\sqrt{}$ sign Not ISW, eg $\sqrt{21.75}$ (or 4.66) M1A0 ans 4.66, no working, M1A0 NB $\frac{1768}{52}-"146.5"^2$ or 1768 -('-3.5^2') M0A0	Σm = 150x52 - 182 or 7618 B1 "7618" ÷ 52 M1 =146.5 A1 (Σ(m-150)²=1768 Σm²-300Σm+150²x52=1768 Σm²=1768+300x"7618"-150²x52 = 1117168) $\frac{1768+300\times'7618'-150^2\times52}{52}-'146.5'^2$ or $\frac{'1117168'}{52}-'146.5'^2$ fully correct method M1 = 21.75 A1	

Question			Answer	Mk	Guidance	
	ii		$\frac{\Sigma m^2}{52}$ –"146.5"^{2} = "21.75" or Σm^2 = ('21.75' + '146.5^2') × 52 ft their mean & +ve var from (i) for M2 Σm^2 = 1117168 ISW	M2 A1 **[3]**	Allow M1 for $\frac{\Sigma m^2}{52}$ –"3.5"^{2} = "21.75" or Σm^2 = ('21.75' + '3.5^2') × 52 Exact; no ft from (i) eg 147 or 21.8	$\Sigma(m-150)^2=1768$ $\Sigma m^2-300\Sigma m+150^2x52=1768$ ≥ 2 terms correct M1 $\Sigma m^2=1768+300x$"7618"-150^2x52 correct method M1 =1117168 A1 Correct ans, no wking M1M1A1 If incorrect ans given with no wking, possibly M1M1 for (ii) may be obtained by correct method seen in (i), However M1M0 or M0M0 is more likely.
3(iii)	The correct method is in the 1st column. However, most candidates will give the allowed method in the middle column and score both marks. NB 3rd column					
	iii		(52 + 1) ÷ 4 = 13.25 or (26+1) ÷ 2 = 13.5 (⇒ 13th apple has mass < 140) ⇒ (no. below 140 =) 13	M1 A1 **[2]**	Allow 52÷ 4 or 26 ÷2 (= 13) M1 ⇒ (no. below 140 =) 13 A1	Allow 52÷ 4 or 26 ÷2 (= 13) M1 (⇒ 13th apple has mass 140) ⇒ (no. below 140 =) 12 A0
	iv		IQR = 15 seen or implied 155+1.5×15 = 177.5 (or > 176) or 140–1.5×15 = 117.5 (or < 130) No outliers	B1 B1 B1 **[3]**	or 22.5 seen or implied 176-155 = 21 (or < 22.5) or 140-130=10 (or < 22.5) Ignore method	$\frac{176-155}{15}$ =1.4 (or <1.5) or $\frac{140-130}{15}=\frac{2}{3}$ (or <1.5) Equivalent correct methods may be seen For 2nd B1 allow 14 ≤ IQR ≤ 16
Total				**13**		
4	i		–1	B1 **[1]**		
	ii		5 pts (or line or curve or zigzag) such that: grad always +ve (not vertical) not in st line.	 B1 B1 dep **[2]**	Allow ≥ 4 pts dep 1st B1 Must be clearly intended not to be st line	eg st line, +ve grad, B1B0 If crosses and curve or line, mark crosses SC Some segments vertical (not all) B0B1

Question			Answer	Mk	Guidance	
	iii		x 1 2 3 4 5 or 1 2 3 4 5 y 1 2 3 5 4 2 1 3 4 5 Allow both sets reversed $\Sigma d^2 = 2$ $r_s = 1 - \frac{6\times"2"}{5(5^2-1)}$ = 0.9	M1 A1 M1 M1 A1 **[5]**	Attempt ranks Correct ranks dep 1st M1 Correct method for Σd^2; ft their ranks NB $\Sigma d^2 = 2^2 = 4$: M0 Sub their Σd^2 into correct formula, dep M1M1, eg not using $\Sigma d^2 = 2^2=4$	$\Sigma x=\Sigma y=15$ $\Sigma x^2=\Sigma y^2=55$ $\Sigma xy=54$ $S_{xx}=S_{yy}=55-(15^2\div5)=10$ $S_{xy}=54-(15^2\div5)=9$ correct method for one S M1 $r_s = \frac{9}{\sqrt{10\times10}}$ fully correct method M1 =0.9 A1 ans 0.9, no wking: full marks ans -0.9 with wking may get M1M1M1 ans -0.9, no wking: no marks
Total				**8**		
5	i	a	0.414(2)	B1 **[1]**		
	i	b	0.4142 – 0.2677 = 0.1465 or 0.147 (3 sf) allow 0.146	M1 A1 **[2]**	$^{25}C_{14} \times 0.4^{11} \times 0.6^{14}$	or their (i) - 0.2677, dep +ve result: M1
	i	c	$25 \times 0.6 \times 0.4$ or 15 - 9 (ie from $np(1-p)$) = 6	M1 A1 **[2]**	Allow $\sqrt{(25 \times 0.6 \times 0.4)}$ or 2.45 for M1	
	ii		$^{24}C_y \times 0.7^{24-y} \times 0.3^y$ oe ($^{24}C_8 \times 0.7^{16} \times 0.3^8$ =) 0.160 (3 sf)	B1 B1 **[2]**	Allow other letters for y Allow 0.16	**NB Must see this for 1st B1** **0.16(0) scores only the second B1** **No M-mark for the correct express'n**
	iii		$(0.8^2)^2 + (2\times0.8\times0.2)^2 + (0.2^2)^2$ oe = $\frac{321}{625}$ or 0.5136 or 0.514	M2 A1 **[3]**	or $0.64^2+0.32^2+0.04^2$ or $\frac{256}{625}+\frac{64}{625}+\frac{1}{625}$ oe	M1 for any correct term or value of term
Total				**10**		

Question			Answer	Mk	Guidance	
6	a		$^{12}C_5$ or $^{12}C_4$ or $^{12}C_3$ oe seen $^{12}C_5 \times {}^7C_4$ ($\times {}^3C_3$) alone, ie correct method = 27720	M1 M1 A1 **[3]**	or $^{12}C_4 \times {}^8C_5$ or $^{12}C_3 \times {}^9C_4$ or etc alone ie any of the six correct products of 2 or 3 C's alone	or $\frac{^{12}P_5}{5!}$ or etc M1 or $\frac{^{12}P_5}{5!} \times \frac{^7P_4}{4!} \times \frac{^3P_3}{3!}$ or other correct M1 or $\frac{12!}{5!4!3!}$ M2
	b	i	$\frac{7!}{3!2!2!}$ or $\frac{^7P_7}{3!2!2!}$ or $\frac{5040}{6\times2\times2}$ oe = 210	M1 A1 **[2]**		
	b	ii	$\frac{1}{210}$ oe or 0.00476 (3 sf)	B1f **[1]**	ft their (b)(i)	Ignore method
	b	iii	$\frac{3\times2\times1\times2}{7\times6\times5\times4}$ or $\frac{12}{840}$ alone M2 or $3\times2\times1\times2$ or $^3C_2\times{}^2C_2\times2\times2$ or 12 oe seen in num or $7\times6\times5\times4$ or 840 oe seen in denom M1 = $\frac{1}{70}$ oe or 0.0143 (3 sf)	M2 A1 **[3]**	$\frac{^3P_2\times{}^2P_2}{^7P_4}$ alone M2 or $^3P_2\times{}^2P_2$ in num or $^7P_4$ in denom M1 $\frac{3}{7}\times\frac{2}{6}\times\frac{1}{5}\times\frac{2}{4}$ oe <u>alone</u> M2 all denoms or all nums correct: M1 But $\frac{3}{7}\times\frac{2}{6}\times\frac{1}{5}\times\frac{2}{4}$ ÷ or × something: M1	$\frac{^3C_2\times{}^2C_2}{^7C_4} \times \frac{1}{(\frac{4!}{2!2!})}$ or $\frac{3}{35}\times\frac{1}{6}$ alone oe M2 or $\frac{^3C_2\times{}^2C_2\times{}^2C_0}{^7C_4}$ or $\frac{3}{35}$ or $\frac{1}{(\frac{4!}{2!2!})}$ or $\frac{4}{^4P_4}$ seen M1 (not $\frac{1}{6}$ alone) NB $^7C_4$ or 35 seen does NOT score, even if in denom
Total				**9**		
7	i		Const prob of scoring oe Each shot indep oe	B1 B1 **[2]**	In context Not 'Prob of goal is <u>consistent</u>' In context Ignore incorrect comments	Prob score on one shot not affected by other shots Each shot indep of previous shot Allow Goals are independent Allow Prob of goals are independent <u>Not</u> Number of goals indep

Question			Answer	Mk	Guidance	
	ii	a	$0.8^2 \times 0.2$ = 0.128 or $\frac{16}{125}$ oe	M1 A1 **[2]**		
	ii	b	$1 - 0.8^9$ = 0.866 (3 sf) (0.865782 . . .)	M1 A1 **[2]**		Long method: all 9 terms correct: M1
	ii	c	$0.8^9 - 0.8^{19}$ or $1 - 0.8^{19} - (1 - 0.8^9)$ or 1 - '0.866' - 0.8^{19} or 1 - 0.8^{19} - '0.866' = 0.120 (3 sf)	M2 A1 **[3]**	Allow M1 for $0.8^{8,\ 9 \text{ or } 10} - 0.8^{18,19 \text{ or } 20}$ or $1 - 0.8^{18,19 \text{ or } 20} - (1 - 0.8^{8,9 \text{ or } 10})$ Allow 0.12	Long method: all 10 terms correct: M2 1 term extra, omitted or incorrect: M1
7(iii)(a) & (iii)(b): SC If 0.2 and 0.3 interchanged, or If 0.3 replaced by $\frac{1}{3}$, consistently throughout (iii)(a) and (iii)(b), all three M-marks can be awarded if consistent working seen OR 'correct' answer with no working. Answers: 0.2 ↔ 0.3: (iii)(a) 0.09 (iii)(b) 0.0364 Use of $\frac{1}{3}$: (iii)(a) 0.04 (iii)(b) $\frac{8}{75}$ or 0.107						
	iii	a	$0.2 \times 0.3 \times 0.2 + 0.2 \times 0.7 \times 0.2$ alone = 0.04 or $\frac{1}{25}$ oe	M1 A1 **[2]**	or 0.2×0.2	
	iii	b	$0.2 \times 0.3 \times 0.8 \times 0.3 + 0.8 \times 0.3 \times 0.2 \times 0.3$ $+ 0.8 \times 0.3 \times 0.8 \times 0.3$ alone oe or $0.3 \times 0.8 \times 0.3 + 0.8 \times 0.3 \times 0.2 \times 0.3$ oe = 0.0864 or $\frac{54}{625}$ oe	 M2 A1 **[3]**	or $(0.2 \times 0.8 \times 2 + 0.8^2) \times 0.3^2$ oe M2 or $\frac{9}{625} + \frac{9}{625} + \frac{36}{625}$ oe M2 or any two correct prods of 4 probs oe :M1 If on tree, must be identified $0.3 \times 0.8 \times 0.3$ M0 unless part of a correct method	$(1 - 0.2^2) \times 0.3^2$ or $(1 - 0.04) \times 0.3^2$ oe M2 or $1 - 0.2^2$ or 1 - 0.04 M1
Total				**14**		

Total 72 marks